

CATALYSIS OF CHIRAL SYMMETRY BREAKING BY EXTERNAL MAGNETIC FIELDS IN THREE DIMENSIONAL LATTICE QED

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The enhancement of the fermionic condensate due to the presence of both homogeneous and non-homogeneous external magnetic fields is studied for three-dimensional QED.

1 Introduction

The reason to study the behaviour of the fermionic matter under the influence of an external magnetic field^{1,2,3,4} has to do with considerations connected to the effects of magnetic fields in the early Universe and to high T_c superconductivity.

The three-dimensional continuum Lagrangian of the model is given by:

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + \bar{\Psi}D_\mu\gamma_\mu\Psi - m\bar{\Psi}\Psi, \quad (1)$$

where $D_\mu = \partial_\mu - ig a_\mu^S - ieA_\mu$; a_μ^S is a fluctuating gauge field, while A_μ represents the external gauge field. The main object of interest here is the condensate $\langle \bar{\Psi}\Psi \rangle$, which is the coincidence limit of the fermion propagator, $S_F(x, y)$. A first estimate of the enhancement of the condensate arising from the external fields may be gained through the analysis of the relevant Schwinger-Dyson equation:

$$S_F^{-1}(p) = \gamma \cdot p - g \int \frac{d^3k}{(2\pi)^3} \gamma^\mu S_F(k) \Gamma^\nu(k, p-k) D_{\mu\nu}(p-k) \quad (2)$$

where Γ^ν is the fermion-photon vertex function and $D_{\mu\nu}$ is the exact photon propagator.

A result which has been obtained⁵ by approximating the Schwinger-Dyson equation for strong magnetic fields is that the dynamically generated mass $\Sigma(0)$

is given by:

$$\Sigma(0) \simeq C \ln \left[\frac{\sqrt{eB}}{\alpha} \right]. \quad (3)$$

There have also been approximations in the regime of smaller magnetic fields with interesting results⁶, but for a fully quantitative treatment one should rely on the lattice approach⁷.

2 Lattice formulation

Let us now describe the lattice formulation of the problem. The lattice action is given by the following formulae:

$$S = \frac{\beta_G}{2} \sum_{x, \mu, \nu} F_{\mu\nu}(x) F^{\mu\nu}(x) + \sum_{n, n'} \bar{\Psi}_n Q_{n, n'} \Psi_{n'} \quad (4)$$

$$F_{\mu\nu}(x) \equiv a_\mu^S(x) + a_\nu^S(x + \mu) - a_\mu^S(x + \nu) - a_\nu^S(x)$$

$$Q_{n, n'} = \delta_{n, n'} - K \sum_{\hat{\mu}} [\delta_{n', n + \hat{\mu}} (r + \gamma_{\hat{\mu}}) U_{n\hat{\mu}} V_{n\hat{\mu}} + \delta_{n', n - \hat{\mu}} (r - \gamma_{\hat{\mu}}) U_{n-\hat{\mu}, \hat{\mu}}^\dagger V_{n-\hat{\mu}, \hat{\mu}}^\dagger].$$

The indices n , n' are triples of integers, such as (n_1, n_2, n_3) , labeling the lattice sites, while μ denotes directions. r is the Wilson parameter, K the hopping parameter, $U_{n\hat{\mu}} \equiv e^{iga\alpha_{n\hat{\mu}}^S}$, $V_{n\hat{\mu}} \equiv e^{ieaA_{n\hat{\mu}}}$, $\beta_G = \frac{1}{g^2 a}$. $\alpha_{n\hat{\mu}}^S$ represents the statistical gauge potential and $A_{n\hat{\mu}}$ the external electromagnetic potential. β_G is related to the statistical gauge coupling constant in the usual way. On the other hand, we denote by e the dimensionless electromagnetic coupling constant of the external electromagnetic field $U_E(1)$. In our treatment we will use naïve fermions, so we set $r = 0$.

3 Lattice Results

We first consider a homogeneous magnetic field and study its effect on the condensate. For the construction of a lattice version of a homogeneous magnetic field we follow⁸. We will not describe the details here, but just say that the magnetic field B is given by the expression $B = m \frac{2\pi}{N^2}$, for a lattice with spatial extent $N \times N$; m is an integer and B runs from 0 to π . We also note that we measured the magnetic field in units of its maximal value: thus we used the parameter b , defined by: $b \equiv \frac{B}{B_{max}}$. Since $B_{max} = \pi$, as explained previously, we get: $b = \frac{B}{\pi}$ and b runs from 0 to 1. We will first present the results for the $T = 0$ case. Figure 1 contains the fermion condensate versus the

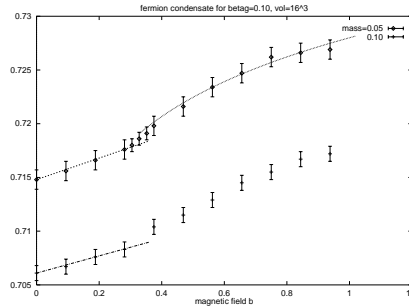


Figure 1: $\langle \bar{\Psi}\Psi \rangle$ versus the magnetic field strength at strong coupling for two masses.

magnetic field for a 16^3 lattice in the strong coupling regime for the statistical gauge field ($\beta_G = 0.10$) for two values of the bare mass. For both masses the plot consists of two parts with qualitatively different behaviour. For b smaller than about 0.3 we find a linear dependence of the condensate on the external magnetic field, while for big magnetic fields we find points that could possibly be fitted to a logarithmic type of curve. The logarithmic dependence is the one referred to above (equation 3) and has been found by an approximate solution of the Schwinger-Dyson equations in the regime of strong magnetic fields ⁽⁶⁾. We have included such a logarithmic guide to the eye for $m = 0.05$ in figure 1.

We now make contact with the results for the model with the statistical gauge field turned off. In ⁷ it has been found that for big enough b the condensate stopped showing a monotonous increase with b , at $b = 0.5$ it had a local minimum and then had a succession of maxima and minima, up to $b = 1$. Moreover, there was a spectacular volume dependence. One expects, of course that this “free” case will be reached for big enough β_G . In figure 2 we show the results for $\beta_G = 0.5$ and $\beta_G = 1.0$ for various volumes. For $\beta_G = 0.5$ the “curve” shows the first sign of “breaking” at $b = 0.5$, while at $\beta_G = 1.0$ the succession of maxima and minima is clear. However, there is no detectable volume dependence, so we can be sure that, even at this large β_G , the limit of switching the gauge field off has not yet been reached; it will presumably be reached for even bigger values of β_G .

Figure 3 contains the zero mass limit of the condensate versus β_G , for four values of the external field. We observe that in the strong coupling region the b -dependence is rather weak; on the contrary, at weak coupling, the condensate is mainly due to the external field and we find an increasingly big b -dependence, as we move to large β_G .

The last topic will be the study of the response of 3-D QED to a non-

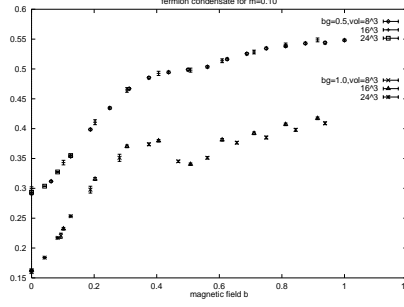


Figure 2: $\langle \bar{\Psi}\Psi \rangle$ versus the magnetic field for two big values of the gauge coupling constant and three volumes.

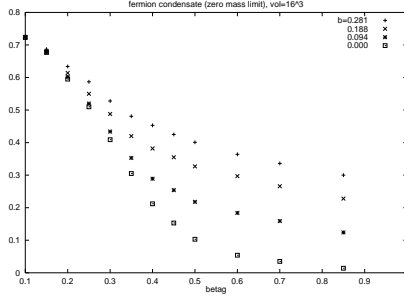


Figure 3: $\langle \bar{\Psi}\Psi \rangle$ versus β_G in the zero mass limit for four values of the magnetic field strength.

homogeneous magnetic field; the lattice approach is very efficient here. The model is considered with the statistical gauge field turned off. For the construction of the non-homogeneous lattice magnetic field, we refer the reader to a forthcoming publication ⁽⁹⁾.

In figure 4 we show the results for a central region of non-vanishing flux of extent 6×6 . More specifically, for the 16^3 lattice we have been using, the region with constant non-zero flux contains the plaquettes starting at (n_1, n_2, n_3) , with $6 \leq n_1 \leq 11$ and $6 \leq n_2 \leq 11$, while n_3 takes all values. Note that nothing depends on the value of n_3 . The uppermost curve in the figure depicts the result for the condensate at the site $(9, 9, 9)$. We have observed that the results for the sites $(9, 9, 9)$, $(9, 10, 9)$, $(9, 11, 9)$, which lie totally within the region of the non-zero flux, are quite similar. The first substantial change takes place at the site $(9, 12, 9)$, shown in the figure, which lies ex-

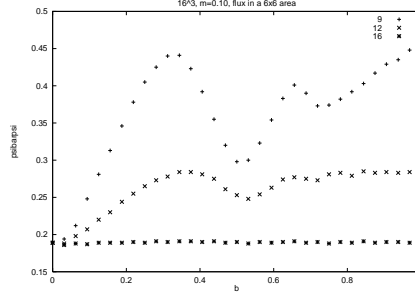


Figure 4: $\langle \bar{\Psi}\Psi \rangle$ versus magnetic field strength where the flux is non zero only in a central region extending over 6x6 plaquettes. The condensate at various distances from the center is shown.

actly on the boundary of the above region. The curves corresponding to the sites (9, 13, 9), (9, 14, 9), (9, 15, 9), (9, 16, 9) dive together to a value which is accounted for by the explicit mass term and has very little to do with the external magnetic field. Only the curve for the last site is shown.

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